d) $m<0$

l'équation $A(\lambda)=m$ n'a pas de solution

$m \in]0,4[$

l'équation $A(\lambda)=m$ admet deux solutions

$m \in \{0\} \cup [4,+\infty[$

l'équation admet une seule solution

EX 9 :

1) $A=\int_0^1 \frac{x}{(1+x^2)^3}dx=\frac{1}{2}\int_0^1 \frac{2x}{(1+x^2)^3}dx$

$=\left[\frac{-1}{4(1+x^2)^2}\right]_0^1=\frac{3}{16}$

2) $A+B=\int_0^1 \frac{x}{(1+x^2)^2}dx=\left[\frac{-1}{2(1+x^2)}\right]_0^1=\frac{1}{4}$

3) $\begin{cases} A=\frac{3}{16} \\ A+B=\frac{1}{4} \end{cases} \Rightarrow B=\frac{1}{16}$

EX 10 :

1) $\int_0^{\frac{\pi}{2}} \sin^2 x dx=\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x)dx$

$=\left[\frac{1}{2}(x-\frac{1}{2}\sin 2x)\right]_0^{\frac{\pi}{2}}=\frac{\pi}{4}$

2) $\int_0^{\frac{\pi}{2}} \cos^2 x dx+\int_0^{\frac{\pi}{2}} \sin^2 x dx=\int_0^{\frac{\pi}{2}} (\cos^2 x+\sin^2 x)dx$

$=\int_0^{\frac{\pi}{2}} dx=[x]_0^{\frac{\pi}{2}}=\frac{\pi}{2}$

$\Rightarrow \int_0^{\frac{\pi}{2}} \cos^2 x dx=\frac{\pi}{2}-\int_0^{\frac{\pi}{2}} \sin^2 x dx$

$\Rightarrow \int_0^{\frac{\pi}{2}} \cos^2 x dx=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

EX 11 :

1) onpose :

$\begin{cases} U'(x)=\cos 2x \xrightarrow{P} U(x)=\frac{1}{2}\sin 2x \\ V(x)=x \xrightarrow{d} V'(x)=1 \end{cases}$

d'où : $\int_0^{\pi} x\cos 2x dx=\left[\frac{1}{2}x.\sin(2x)\right]_0^{\pi}-\int_0^{\pi}\frac{1}{2}\sin 2x dx$

$\int_0^{\pi} x\cos 2x dx=0+\left[\frac{1}{4}\cos 2x\right]_0^{\pi}=0$

2) a)

* $I+J=\int_0^{\frac{\pi}{2}} x\cos^2 x dx+\int_0^{\frac{\pi}{2}} x\sin^2 x dx$

$=\int_0^{\frac{\pi}{2}} x(\cos^2 x+\sin^2 x)dx=\int_0^{\frac{\pi}{2}} x dx$

$=\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{2}}=\frac{\pi^2}{8}$

* $I-J=\int_0^{\frac{\pi}{2}} x(\cos^2 x-\sin^2 x)dx=\int_0^{\frac{\pi}{2}} x\cos 2x dx=0$

b) $\begin{cases} I+J=\frac{\pi^2}{8} \\ I-J=0 \end{cases}$ d'où $I=J=\frac{\pi^2}{16}$

EX 12 :

1) $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}=[tgx]_0^{\frac{\pi}{4}}=1$

2) $f(x)=\frac{\sin x}{\cos^3 x}$; $x \in \left[0,\frac{\pi}{4}\right]$

$f'(x)=\frac{\cos x.\cos^3 x-\sin x(-3\sin x\cos^2 x)}{\cos^6 x}$

$=\frac{\cos^4 x+3\sin^2 x.\cos^2 x}{\cos^6 x}=\frac{1}{\cos^2 x}+\frac{3\sin^2 x}{\cos^4 x}$

$=\frac{1}{\cos^2 x}+\frac{3(1-\cos^2 x)}{\cos^4 x}=\frac{1}{\cos^2 x}+\frac{3}{\cos^4 x}-\frac{3}{\cos^2 x}$

$f'(x)=\frac{3}{\cos^4 x}-\frac{2}{\cos^2 x}$

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